Theorem 1  An edge $e$ of a connected graph $G$ is a bridge if and only if it lies on no cycle.

Proof.  We will begin by showing that if $e$ is a bridge, then it is on no cycle. We will prove the contrapositive, that if $e$ is on a cycle, then it is not a bridge.

Let $e = sv$. If $e$ lies on the cycle $s, v, v_1, \ldots, v_k, s$, then $v, v_1, \ldots, v_k, s$ is an $s-v$ path in $G - e$, so $e$ is not a bridge.

Next, we need to show that if $e$ is not on a cycle, then it is a bridge. Once again, we can prove the contrapositive, which is that if $e$ is not a bridge, then it lies on a cycle. Because $e$ is not a bridge, we know that $G - e$ is still connected, and in particular, that there is an $sv$ path in $G - e$. That path together with the edge $e$ forms a cycle in $G$.  

\[\blacksquare\]