Work from 2011-10-13

MCS-236 class

October 13, 2011

‘

Theorem 1 *An edge e of a connected graph G is a bridge if and only if it lies on no cycle.*

Proof. We will begin by showing that if e is a bridge, then it is on no cycle. We will prove the contrapositive, that if e is on a cycle, then it is not a bridge.

Let $e = sv$. If e lies on the cycle $s, v, v_1, \ldots, v_k, s$, then $v, v_1, \ldots, v_k, s$ is an $s - v$ path in $G - e$, so e is not a bridge.

Next, we need to show that if e is not on a cycle, then it is a bridge. Once again, we can prove the contrapositive, which is that if e is not a bridge, then it lies on a cycle. Because e is not a bridge, we know that $G - e$ is still connected, and in particular, that there is an sv path in $G - e$. That path together with the edge e forms a cycle in G. ∎