1 Show that a graph $G$ is regular if and only if $\overline{G}$ is regular.

Proof. [direct]  a.) Show that a graph $G$ is regular if $\overline{G}$ is regular. If $\overline{G}$ is regular, then all vertices in $\overline{G}$ have a degree of $r$, where $0 \leq r \leq n - 1$. By definition, every edge $xy$ of $\overline{G}$ is not in $E(G)$, and every edge $x_1y_1$ of $G$ is not in $\overline{G}$. So since every vertex $v$ in $\overline{G}$ has a degree of $r$, $v$ is not adjacent to any of the $r$ vertices to which it is adjacent in $\overline{G}$. But in $G$, $v$ is also adjacent to any vertices not contained in its set of $r$ neighbors in $\overline{G}$. However, $v$ cannot be adjacent to itself either, so in $G$, $v$ has $n - r - 1$ additional neighbors. Thus, the degree of $v$ in $G$ is $r - r + n - r - 1 = n - r - 1$. Therefore, every vertex $v$ in $G$ has a degree of $n - r - 1$, so $G$ is regular.

b.) Show that a graph $\overline{G}$ is regular if $G$ is regular. If $G$ is regular, then all vertices in $G$ have a degree of $r$, where $0 \leq r \leq n - 1$. By definition, every edge $xy$ of $G$ is not in $E(\overline{G})$, and every edge $x_1y_1$ of $\overline{G}$ is not in $G$. Thus by the same logic as in (1.1), $\overline{G}$ is regular if $G$ is regular.

2 Show that if $G$ and $\overline{G}$ are both $r$-regular for some nonnegative integer $r$, then $G$ has odd order.

Proof. [direct]

If $G$ is regular for some nonnegative integer $r$, then by HW 2.26 [1], every vertex in $\overline{G}$ must have a degree of $n - r - 1$, where $n$ is the order of $G$ and $\overline{G}$. Since $\overline{G}$ is also $r$-regular, $n - r - 1 = r$, so $n = 2r - 1$. Since $r$ is an integer, $n$ is odd. Therefore, $G$ has odd order.