PHY 370 – Advanced Mathematical Methods

Spring 2024

Instructor:	Rafid Mahbub	**Time:**	M T W Th F 12:30 – 1:20 pm
Email:	mahbub@gustavus.edu	**Place:**	Olin 216

Course Pages: Moodle class page

Office Hours: I am flexible with office hours. You can drop by Olin 212 anytime as long as I am in the office. I usually take lunch break from 12:30 to 1:30 pm. So there is a big chance that I will not be available during that one hour.

Main references: The main references for this course are

- Mathematical Methods for Scientists and Engineers – Donald A. McQuarrie
  This textbook has been used for this course over the last few iterations. It is a well-written book and covers the materials we require at the desired level.
- Mathematical Methods for Physicists – George B. Arfken, Hans J. Weber
  Although more advanced in many areas, this is one of the best textbooks for mathematical methods as applied to the field of physics. Despite this, it is recommended reading.

Other references: Some other references you should consider are

- Mathematical Methods in Engineering and Physics – Gary N. Felder, Kenny M. Felder
  This is used by Tom for PHY 250 and is very good for review of more elementary materials.
- Schaum's Mathematical Handbook of Formulas and Tables – John Liu, Murray R. Spiegel, Seymour Lipschutz

Objectives: This course is aimed at an introduction to some advanced mathematical techniques that are essential requirements in the training of young physicists and engineers, some of which you may have already encountered in previous physics courses without the time to go through them in any detail. Although the potential list of mathematical topics is vast, this course will primarily focus on two areas that will be heavily used in future college and graduate school courses. In fact, this course should be counted as prerequisite for enrolling in PHY 390 (quantum mechanics). These two topics are

- Linear algebra: Our focus will be at understanding and generalizing concepts of linear vectors spaces and vectors and operators defined on such vector spaces. As such, the aim is to expand our notions of vectors and operators as being finite dimensional column and square matrices to more general objects which, nonetheless, follow the axioms of linear algebra. This includes an understanding of how functions and polynomials defined within an interval can behave as vectors and how derivatives behave as differential operators. Furthermore, we will learn how to use Dirac's 'bra-ket' notation for denoting vectors and their various operations.
- Differential equations: Perhaps the biggest focus of this course is the study and application of differential equations – both ordinary and partial. We will perform a systematic study of second order ODEs, exploring their series solutions, often arriving at special functions and polynomials. Sturm-Liouville theory and Green function will also be discussed in some

detail. Considerable time will be spent on understanding PDEs that frequently appear in physics and solution techniques using separation of variables, Fourier transform and Green function techniques. Much like the linear algebra part of this course, these topics in differential equations will play an equally important role in preparation for quantum mechanics.

Prerequisites: It is expected that you have taken PHY 250 (or equivalent preparation) where you studied differential equations, linear algebra and vector calculus. Although reviews will be performed where needed, these will be rather brief since this course will explore advanced topics in these fields. Moreover, some familiarity with numerical computation will be assumed since homework problems will often ask you to write small pieces of codes to confirm analytical calculations and produce plots.

Tentative Course Outline: The following is only a tentative outline of the course. Completion of the different sections each week will depend on many factors.

- **Week 1 (02/05 – 02/09)**
  - **Infinite series:** types of infinite series, their algebra and tests of convergence; power series
  - **Special integrals:** gamma and beta functions; Gaussian integrals; volume and surface area of n-sphere
- **Week 2 (02/12 – 02/16)**
  - wrapping up special integrals with error functions and introduction to asymptotic techniques; Laplace's method
  - **Dirac delta distributions:** Dirac delta distributions as a limiting case of sequence of functions, properties
  - **Linear Algebra:** review of elementary concepts, index notation and Einstein summation convention, Dirac's 'bra-ket' notation
- **Week 3 (02/19 – 02/23)**
  - **Linear Algebra:** linear vector spaces, Gram-Schmidt procedure, linear transformations (Hermitian and unitary matrices), eigenvalue problems, examples of eigensystems encountered in physics
- **Week 4 (02/26 – 03/01)**
  - **Linear Algebra:** Kronecker delta and Levi-Civita symbol, Einstein summation convention, change of basis and diagonalization (orthogonal and unitary transformations), elements of group theory with SO(3) and SU(2)
- **Week 5 (03/04 – 03/08)**
  - **Ordinary differential equations:** survey of elementary techniques for solving first-order and second-order linear ODEs, the Wronskian and its application to solving homogeneous linear ODEs

- **Week 6 (03/11 – 03/15)**
  - **Ordinary differential equations:** series solution of differential equations and Frobenius method; Legendre differential equation; singularities and their classification; series solution in the presence of singularities
- **Week 7 (03/18 – 03/22)**
  - **Ordinary differential equations:** Bessel differential equation; Bessel functions
  - **Asymptotic methods:** more on asymptotic expansions, asymptotic methods for ODEs
- **Week 8 (03/25 – 03/29)**
  - Spring break
- **Week 8 (04/01 – 04/05)**
  - **Ordinary differential equations & asymptotic methods:** perturbation methods for ODEs, Poincaré-Lindstedt method and method of multiple scales, WKB method
- **Week 9 (04/08 – 04/12)**
  - **Ordinary differential equations:** orthogonal polynomials and functions (orthonormality, recursion relations and generating functions), Sturm-Liouville theory, Green function techniques
- **Week 10 (04/15 – 04/19)**
  - **Fourier techniques:** Fourier series, convergence of Fourier series, Gibbs phenomenon, applications to ODEs
- **Week 11 (04/22 – 04/26)**
  - **Partial differential equations:** survery of types and occurrence of PDEs in physics (the importance of $\boldsymbol{\nabla}$ operator in physical systems)
  - Laplace equation, uniqueness of solutions, solution in Cartesian coordinates using Fourier series
  - wave equation in 2D and 3D
- **Week 12 (04/29 – 05/03)**
  - **Partial differential equations:** heat and diffusion equations, diffusion as a continuous limit of random walk, some comments on Markov processes
  - **Fourier techniques:** extending Fourier series no aperiodic functions, Fourier transform and its inverse; Parseval's identity

- **Week 13 (05/06 – 05/10)**
  - **Fourier techniques & partial differential equations:** continuation of Fourier transforms; convolutions; stationary phase method; solution of ODEs and PDEs with Fourier transforms
  - Green function techniques as applied to solutions of PDEs
- **Week 14 (05/13 – 05/17)**
  - additional topics

Grading and evaluation: The grading scheme for the course is: problem sets (30%), midterm (30%) and take home final (40%). The letter grade boundaries are: A ($>$ 90%), A- ($\geq$ 85% to $<$ 90%), B+ ($\geq$ 80% to $<$ 85%), B ($\geq$ 75% to $<$ 80%), B- ($\geq$ 70% to $<$ 75%), C+ ($\geq$ 65% to $<$ 70%), C ($\geq$ 60% to $<$ 65%), C- ($\geq$ 55% to $<$ 60%), D+ ($\geq$ 50% to $<$ 55%), D ($\geq$ 45% to $<$ 50%) and F ($<$ 45%)

The midterm exam will be held on week 7 during class hours. The final exam will be take home. Upon deciding on a date, the final exam questions will be made public in Moodle. You will be able to work on the solutions the entire day, being able to use additional resources like your notes and previous problem sets. The solutions should be turned in to my box in the mailroom the following day by 10 am. The final exam problems will be similar to homework problem sets. But there will be multiple problems given that you will have an entire day to finish writing the solutions.

Attendance: You are expected to regularly attend all the classes. You will be responsible for missing important announcements/deadlines in the event of absences.

Incomplete: In the event of extenuating circumstances preventing you from meeting homework submission deadlines or sitting for exams, you will be given an incomplete grade.

Academic Honesty: Having signed and agreed to abide by the College's Honor Code, students thereby pledge that, in all academic exercises, examinations, papers, and reports, they shall submit their own work. In the context of this course, students are expected to collaborate and to discuss their out-of-class assignments. However, submitting under one's own name work that is merely copied from another is a violation of the Honor Code. Furthermore, seeking outside assistance during exams is expressly forbidden. A full description of the Academic Honesty Policy and the Honor Code can be found in the Academic Catalog.

Requesting Accomodations: Gustavus Adolphus College is committed to ensuring equitable and inclusive learning environments for all students. If you have a disability and anticipate or experience barriers to equal access, please speak with the accessibility resources staff about your needs. A disability may include mental health, attentional, learning, chronic health, sensory, physical, and/or short-term conditions. Students with a documented elevated risk of COVID-19 may also request academic accommodations. When appropriate, staff will guide students and professors in making accommodations to ensure equal access. Accommodations cannot be made retroactively; therefore, to maximize your academic success at Gustavus, please contact them as early as possible. Accessibility resources staff are located in the Academic Support Center (`https://gustavus.edu/asc/accessibility/`). Accessibility Resources Coordinator, Katy Clay, (clayk@gustavus.edu), can provide further information.

Mental Wellbeing: The Gustavus community is committed to and cares about all students. Strained relationships, increased anxiety, alcohol or drug problems, feeling down, difficulty concentrating, and/or lack of motivation may affect a student's academic performance or reduce a student's ability to participate in daily activities. If you or someone you know expresses such mental health concerns or experiences a stressful event that can create barriers to learning, Gustavus services are available to assist you, and include online options. You can learn more about the broad range of confidential health services available at https://gustavus.edu/counseling/ and https://gustavus.edu/deanofstudents/services/.