# MCS-236 Homework Exercise 2.26

Student Two

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(a):

**Proof.** The complete graph of $G$ is $G \cup \overline{G}$. Let $n$ is the order of $G$. So every vertex in the complete graph has $n-1$ degrees. Suppose $G$ is a $k$-regular graph, then every vertex in $G$ has $k$ degrees. So we have every vertex in $\overline{G}$ has $n-1-k$ degrees. Since $n$ and $k$ are constants, $n-1-k$ is constant. So we have $\overline{G}$ is a $(n-1-k)$-regular graph. So $\overline{G}$ is regular if $G$ is regular. And we can proof the other way around by the same method. Thus, $G$ is regular if and only if $\overline{G}$ is regular. ∎

(b):

**Proof.** Let the $n$ is the order of $G$. Since $G$ and $\overline{G}$ are both $r$-regular graph, every same vertex in $G$ and $\overline{G}$ share the degrees this vetex in the complete graph of $G$ has equally. That is to say every vetex in $G$ and $\overline{G}$ has $(n-1)/2$ degrees, which equal to $r$. Because $r$ is a nonnegative integer, $(n-1)/2$ is a nonnegative integer too. So $n$ has to be an odd number. Thus, if $G$ and $\overline{G}$ are both $r$-regular for some nonnegative integar $r$, then $G$ has odd order. ∎