HW 2.26

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1 Show that a graph G is regular if and only if $\overline{G}$ is regular.

Proof. [direct] a.) Show that a graph G is regular if $\overline{G}$ is regular. If $\overline{G}$ is regular, then all vertices in $\overline{G}$ have a degree of r, where $0 \leq r \leq n-1$. By definition, every edge xy of $\overline{G}$ is not in $E(G)$, and every edge x_1y_1 of G is not in $\overline{G}$. So since every vertex v in $\overline{G}$ has a degree of r, v is not adjacent to any of the r vertices to which it is adjacent in $\overline{G}$. But in G, v is also adjacent to any vertices not contained in its set of r neighbors in $\overline{G}$. However, v cannot be adjacent to itself either, so in G, v has $n-r-1$ additional neighbors. Thus, the degree of v in G is $r-r+n-r-1=n-r-1$. Therefore, every vertex v in G has a degree of $n-r-1$, so G is regular.

b.) Show that a graph $\overline{G}$ is regular if G is regular. If G is regular, then all vertices in G have a degree of r, where $0 \leq r \leq n-1$. By definition, every edge xy of G is not in $E(\overline{G})$, and every edge x_1y_1 of $\overline{G}$ is not in G. Thus by the same logic as in (1.1), $\overline{G}$ is regular if G is regular.

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2 Show that if G and $\overline{G}$ are both r-regular for some nonnegative integer r, then G has odd order.

Proof. [direct]

if G is regular for some nonnegative integer r, then by HW 2.26 [1], every vertex in $\overline{G}$ must have a degree of $n-r-1$, where n is the order of G and $\overline{G}$. Since $\overline{G}$ is also r-regular, $n-r-1=r$, so $n=2r-1$. Since r is an integer, n is odd. Therefore, G has odd order.

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