MCS-236 Non-textbook Homework Exercise 12

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The statement of Theorem 10.10 on page 276 includes the premise $k \geq 3$. That premise is unnecesarily restrictive.

1. Give an example showing that the theorem's conclusion holds also for $k = 2$.

2. Can the premise be loosened to $k \geq 1$ or only to $k \geq 2$? Either give an example with $k = 1$ or explain why none exists.

Note that the book's proof treats $k = 3$ and $k = 4$ as both being base cases and uses the inductive case only for $k \geq 5$. This isn't necessary; you should be able to convince yourself that the inductive case would work fine for $k = 4$, allowing the base case to be limited to $k = 3$. The next question is whether this same process can be extended even further.

3. Start with the smallest example graph you found in the parts 1 and 2 of this homework. Label that graph with a minimum coloring. Apply to this graph the construction described in the proof's inductive case so as to produce a larger graph. Label the vertices of this larger graph with a coloring produced as described in the portion of the proof starting "We can extend ...."

4. Is the coloring of the larger graph a minimum coloring?

5. Is the larger graph triangle free?

6. How would you edit the book's proof to make it cover as broad a range of k values as possible and use as few base cases as possible?