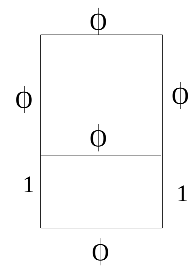
MCS-115

Answers to Homework: 4.3, 4.5, 4.7

4.3.12 Note that after we add the square, we have a rectangle with the following property: If we remove the largest square, we are left with a Golden Rectangle. Though not proved in the text, the Golden Rectangle is the **only** rectangle that has this feature, and so the original rectangle is indeed a Golden Rectangle. If the original sides had length 1 and ϕ, then the new rectangle has sides of length $1+\phi$ and ϕ. Verify by calculator or by algebra that $(1+\phi)/\phi = \phi$ to prove that the new rectangle is also golden.

Remember that $\frac{1}{\phi-1} = \frac{\phi}{1}$, so $\phi - 1 = \frac{1}{\phi}$.

$$\frac{1+\phi}{\phi} = \frac{1}{\phi} + \frac{\phi}{\phi} = \frac{1}{\phi} + 1 = (\phi-1)+1 = \phi - 1 + 1 = \phi$$

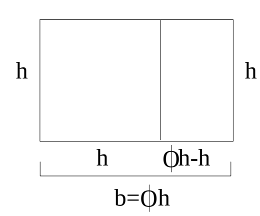


4.3.13 Note that the new longer side in each case is the sum of a longer side and a shorter side of the previous rectangle. As the rectangles increase in size, you should find the longer side divided by shorter side gets closer and closer to the Golden Ratio.

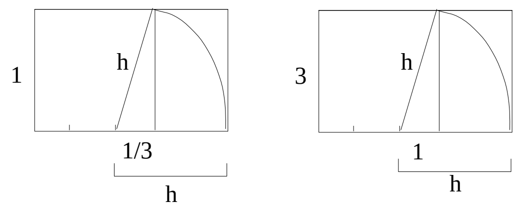
4.3.16 Let b and h denote the base and height of the larger Golden Rectangle. We'll assume that b is the longer side, so $b/h = \phi = \frac{1+\sqrt{5}}{2}$. We can write $b = \phi h$. The area of G is then $\phi h \cdot h = \phi h^2$. Since we get the smaller Golden Rectangle by removing the largest square possible from G, we know that the longer side of the smaller rectangle is also h. The length of the shorter side of G' is $\phi h - h$. The area of G' is $h(\phi h - h)$. The ratio of the areas is

$$\frac{\text{Area}(G)}{\text{Area}(G')} = \frac{\phi h^2}{h(\phi h - h)} = \frac{\phi h^2}{h^2(\phi - 1)} = \frac{\phi}{\phi - 1} = \phi\left(\frac{1}{\phi - 1}\right) = \phi^2$$

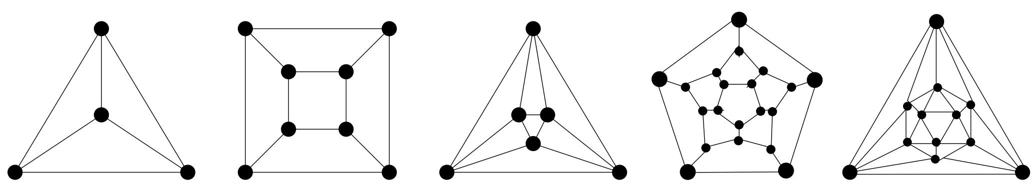
This doesn't depend on the base and height of the original rectangle.



4.3.17 Let the sides of the square be 1 unit long. The right triangle has base 1/3 and height 1, its hypotenuse has length $\sqrt{10/9} = \sqrt{10}/3$. The base of the large rectangle has length $\frac{2}{3} + \frac{\sqrt{10}}{3}$. The ratio of longer side to shorter side for the larger rectangle is $\frac{2}{3} + \frac{\sqrt{10}}{3} \approx 1.720$. The smaller rectangle has sides 1 and $\frac{2}{3} + \frac{\sqrt{10}}{3} - 1 = \frac{\sqrt{10}-1}{3}$. The ratio of longer side to shorter side for the smaller rectangle is $\dfrac{1}{\frac{\sqrt{10}-1}{3}} = \dfrac{3}{\sqrt{10}-1} \approx 1.387$. The ratios are not the same. You could also start with the sides of the square 3 units long. This simplifies some of the calculations.



4.5.8

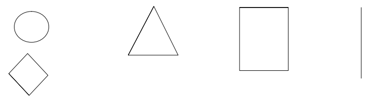


Tetrahedron Hexahedron (Cube) Octahedron Dodecahedron Icosahedron

4.5.15 F=24, V=14, E=36
The new solid has (6 faces of the cube · 4 glued faces) = 24 faces. The new solid has (8 original vertices of the cube + 1 new vertex for each of the 6 faces of the cube) = 14 vertices. The new solid has (12 original edges of the cube + 4 new edges for each of the 6 faces) = 36 edges. Notice that $V - E + F = 14 - 36 + 24 = 2$.

4.5.16 Slicing off a vertex generates a triangle because the cutting plane intersects three sides of the cube. If we continue making parallel cuts, that triangle will get larger until additional sides intersect the cutting plane. At this point the slices will look like triangles with one or more vertices cut off. If your cuts generate equilateral triangles, then when you are halfway through the cube, the slice will be a hexagon. Depending on where you cut, you can get a wide variety of 3,4,5 and 6 sided shapes.

4.7.6 Object 1: resembles (hollow) circle or (hollow) diamond
Object 2: a hollow triangle, base first
Object 3: a hollow square or rectangle, base first
Object 4: a line



4.7.7 Object 1: boundary of a sphere (hollow) or two cones on top of one another (hollow)
Object 2: (hollow) tetrahedron, point first
Object 3: capped off cylinder with bulge in middle or hollow sphere with poles cut off
Object 4: a donut (hollow) or an inner tube

4.7.9 Cases 3 and 4 require tearing in order to get the gold; cases 1 and 2 do not.

4.7.12 The pictures left to right have four, five, and six crossings respectively. At each crossing one portion of the rope passes under another portion. We can invert any crossings we want using the fourth dimension. In general, take a small portion of one of the knots and push it into the fourth dimension. If we look only at the original three dimensions, it appears as if the rope were cut. Take the other piece of rope and slip it past this hole to switch the crossing. Finally, push the small portion of rope back into the original three-dimensional space.

4.7.16

Dimension	V	E	2D faces	3D faces
1	2	1	0	0
2	3	3	1	0
3	4	6	4	1
4	5	10	10	5
5	6	15	20	15
n	n+1	$E_{n-1} + V_{n-1} = 1 + 2 + \cdots + n =$ n(n-1)/2	$2DF_{n-1} + E_{n-1}$	$3DF_{n-1} + 2DF_{n-1}$

In each case, all the vertices are connected to all the other vertices. The number of edges is the number of edges in the previous dimension plus the number of vertices in the previous dimension.