MCS-115

Answers to Homework: 7.2, 7.3, 7.5

7.2.7 Completely fulfilling all promises (0.00001), Picking the Queen of Hearts (1/52), Seeing a full moon (1/30), Picking an ace (1/13), Rolling a 6 (1/6), Picking a black card (1/2), Flying safely (0.9999999). Obviously, some of these are non-rigorous guesses at the probabilities.

7.2.10 17 letters, 4 Rs, 2 Bs, 9 in the first half (A-M) and 5 vowels. There is a 4/17 chance of getting an R, 2/17 chance of getting a B, 9/17 chance of pulling a letter from the first half of the alphabet and 5/17 chance of pulling a vowel.

7.2.14 There are 3 numbers with 3 as a factor (3,6,9), 4 prime numbers (2,3,5,7), 5 even numbers, and 0 numbers evenly divisible by 13. So the corresponding probabilities are 3/10, 4/10=2/5, 5/10=1/2, and 0/10=0 respectively.

7.2.15

Penny	H	H	H	H	T	T	T	T
Nickel	H	H	T	T	H	H	T	T
Dime	H	T	H	T	H	T	H	T

Three presidents? 1/8. Exactly two presidents? 3/8. If you only know that a head is showing, then you can rule out the (T,T,T) possibility so that there are really only 7 equally likely outcomes. Now the probability of three presidents is 1/7. Knowing that Lincoln is showing eliminates four the of the equally likely outcomes. Given this, the probability of seeing three heads is reduced to 1/4.

7.2.19 Answer: $(1/365)^2$. Let's call the two people Tip and Kirk. The chance that Tip was born on December 9 is 1/365 (forget leap year). The chance that Kirk was born on this day is also 1/365. The chance that both Tip and Kirk were born on this day is (1/365)(1/365).

7.2.20 Answer: $\frac{30}{36} = \frac{5}{6}$. It is easier to find the probability that the sum doesn't exceed 4. The only such sums are (1-1), (1-2), (2-1), (1-3), (3-1), and (2-2). The chance of not exceeding 4 is 6/36=1/6, so the chance of exceeding 4 is $1 - \frac{6}{36} = \frac{30}{36}$.

7.2.24 Answer: 0.109. The chance that first 40 people don't match is $\frac{365 \cdot 364 \cdot 363 \cdots 326}{366^{40}} \approx 0.109$. You could also use the information in the text on page 533. The probability that there is a match when there are 40 people is .891. The probability that there isn't a match is $1 - .891 = 0.109$.

7.2.30 This is somewhat non-intuitive. The probability that the other side is blue is 1/3. Remember that our notions of probability stem from manipulating sets of *equally likely* outcomes. Red-red, red-blue, blue-red, and blue-blue are not all equally likely outcomes. Assume that all the cards and colors are different, for example, (pink,red), (blue-navy), (ruby-red, sky-blue). You are shown a side of a card with one of the red hues. It might be the pink side, the red side, or the ruby-red side. Among these three equally likely possibilities, in two of them, in two of them the other side is another red hue and in only one is the other side a blue hue. So the probability that the the other side is blue is 1/3.

7.3.18 Answer: 9. After the first week, 300 people see a perfect record; second week, 150; third week, 75. Since 75 is odd we have to break the symmetry (75=38+37). If we are lucky, 38 people will still see a perfect track record. If we are always lucky, 2 people will see a perfect track record for 9 consecutive weeks: 300-150-75-38-19-10-5-3-2.

7.3.21 With a 1/4 chance of correctly answering each question, the chance of a perfect score is $(1/4)^{100}$ or approximately 10^{-60}. With educated guesses, the chance goes up to $(1/2)^{100}$, or roughly 10^{-30}, still vanishingly small. Note that the chances of getting them all wrong are similarly small. The first guy will score close to 25% while the second guy with score closer to 50%.

7.3.26 Answer: $(1/26)^3$. Chance of typing 'c' first is 1/26, and the chance of then typing an 'a' is also 1/26, and the chance of typing the final 't' is also 1/26. Since all three independent events must occur, we multiply the probabilities. $(1/26)^3 = 0.00005\ldots$

7.3.28 Answer: about 67%. The chance of missing the number on any given day is 999/1000. The chance of guessing wrong every day for 3 years is $(999/1000)^{3(365)}$, and so the chance of getting at least one correct number is $1 - 0.999^{1095} \approx .67$.

7.3.30 Since EDWAR... has 12 letters and MICHA... has 15 letters, EDWAR... will appear much more frequently (roughly 26^3 times more frequently). The chance of typing MICHA... right off the bat is $(1/26)^{15}$, and since you effectively start over with each key stroke, you will see Michael's full name roughly once every 26^{15} digits.

7.3.31 Both outcomes are equally unlikely; they both occur with probability$(1/2)^{10} = 0.001$. The second outcome seems more random because it doesn't have an easily describable pattern like HTHTHTHTHT or HHHHHTTTTT, for example. We might also naturally describe the first outcome as (10 heads, 0 tails) and the second as (6 heads, 4 tails). The chance of rolling (6 heads, 4 tails) is 0.205 while the chance of rolling (10 heads, 0 tails) is still 0.001 which gives a sense of why we think the second outcome should be more common.

7.5.1 Answer: 8. Suppose there were H non-cheaters in your group. The only No's will come from these H people when they flip tails. Since roughly half of them will flip tails, you expect H/2 No's. Since there are 46 No's, solve H/2=46 to estimate the number of honest students. H=92, leaving 8 cheaters in the group.

7.5.7 If we use the one-coin method to survey 1000 students, we expect that 500 of these students will flip heads and answer Yes. Of the 380 students who like to watch cartoons, we can expect half, or 190 students, to flip heads and be amoung the 500 students we are already counting as answering Yes. The other 190 students who like to watch cartoons can be expected to flip tails and also answer Yes. Therefore, we can expect 500 + 190, or 690, students to answer Yes.

7.5.10 Answer: 10. Suppose there were K kissers who flipped two heads and N non-kissers who flipped two heads. We assume that the numbers are the same for the other three groups (HT, TH, TT) and that the total number of kissers is 4K. We expect N+3K people to answer yes and K+3N people to answer no. So N+3K=50 and K+3N=130.

$$\begin{aligned} N &= 50 - 3K \\ K &= 130 - 3N \\ K &= 130 - 3(50 - 3k) \\ K &= 130 - 150 + 9K \\ 8K &= 20 \\ K &= 20/8 \\ 4K &= 10 \end{aligned}$$

7.5.17 The very cooperative Saturday morning group will, on average, study far more hours than the crowd of students at the party. The mean, median and mode will be shifted toward zero in the 9:00pm group.