MCS-115
Answers to Homework: 5.3

5.3.7 Note that the graphs as drawn are not planar since there is a missing vertex where the three edges meet towards the bottom of the graph.

graph	V	E	F
original	7	8	3
next	7	9	4
next	8	9	3
next	7	9	4
next	8	9	3

The original graph has 7 vertices, 8 edges, and 3 faces $(7 - 8 + 3 = 2)$.
Next adds 1 edge and 1 face.
Next adds 1 edge and 1 vertex.
Next adds 1 edge and 1 face.
Next adds 1 edge and 1 vertex.
In either case we get a new edge and a new vertex or a new edge and a new face. In both cases the formula $V - E + F = 2$ remains valid.

5.3.9 You can not reduce the number of faces in a graph by adding edges. If the new edge is created by connecting two previous vertices, then one new region is added. If the new edge is created by adding one vertex and connecting it to an old vertex, then no new regions are created. In either case, the formula $V - E + F$ remains unchanged. There is no other way to add an edge to a graph.

5.3.15 Use $V - E + F = 2$. If $E = 36$ and $F = 18$, then $V = 2 + E - F = 20$. There are 20 vertices.

5.3.16 If F is odd, and V and E are both even, then $V - E + F = 2$ won't work. $V - E$ is even, F is odd, and an even plus an odd is odd; but 2 isn't odd. This contradiction shows that no such connected graph exists.

5.3.21 There are 13 vertices, 24 edges, and 13 faces implied by this 2-dimensional drawing.
$V - E + F = 13 - 24 + 13 = 2$. If we looked at this as a 2D graph, we have instead $V = 12$, $E = 18$, and $F = 8$ (count the outside, too). Again, $V - E + F = 2$.

5.3.26 The left piece has eight vertices, nine edges, and three faces (counting the outside face). Note, $8 - 9 + 3 = 2$. The right piece has six vertices seven edges, and three faces. Again, we have $6 - 7 + 3 = 2$. The composite graph has $8 + 6 = 14$ vertices, $9 + 7 = 16$ edges, and $3 + 3 - 1 = 5$ faces (because we counted the outside face twice). So, $V - E + F = 14 - 16 + 5 = 3$, the magic number for a two-component graph.

5.3.29 This polygonal torus has 9 vertices, 18 edges, and 9 faces. $V - E + F = 9 - 18 - 9 = 0$, the magic number for a polygonal torus.