Homework 8

MCS-375

Fall 2011

Exercise 24.3-2 (3 points)

This exercise appears on page 663. Your example should include not only a weighted directed graph, but also an indication of which vertex is the source. In order to make a convincing case that Dijkstra's algorithm produces incorrect answers, you should choose an example where correct answers are well defined.

Exercise 24.3-10 (3 points)

This exercise appears on page 664.

Exercise 25.2-x1 (3 points)

In order to calculate the Π matrix, the Floyd-Warshall algorithm on page 695 can be modified by incorporating Equations 25.6 and 25.7 as shown on the next page of this homework. In order to make it clear where the new material has been inserted, lines retained from the book's version are shown with their original line numbers; the newly inserted lines have fractional line numbers. The **return** statement on line 8 was also augmented.

Floyd-Warshall(W)

$n = W.rows$
let $\Pi^{(0)} = \left(\pi_{ij}^{(0)}\right)$ be a new $n \times n$ matrix
for $i = 1$ **to** n
    for $j = 1$ **to** n
        if $i == j$ **or** $w_{ij} == \infty$
            $\pi_{ij}^{(0)} = \text{NIL}$
        else
            $\pi_{ij}^{(0)} = i$
$D^{(0)} = W$
for $k = 1$ **to** n
    let $D^{(k)} = \left(d_{ij}^{(k)}\right)$ be a new $n \times n$ matrix
    for $i = 1$ **to** n
        for $j = 1$ **to** n
            if $d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$
                $\pi_{ij}^{(k)} = \pi_{ij}^{(k-1)}$
            else
                $\pi_{ij}^{(k)} = \pi_{kj}^{(k-1)}$
            $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$
return $D^{(n)}$, $\Pi^{(n)}$

Exercise 25.2-4 (p. 699) points out that the superscripts can be dropped from the D matrix in the Floyd-Warshall algorithm, resulting in the Floyd-Warshall′ algorithm shown in that exercise. The returned D matrix is the same and the memory consumption is reduced. This leads to the question of whether the same trick can be played with the Π matrix. Our modified Floyd-Warshall′ would look as shown on the next page of this homework.

FLOYD-WARSHALL$'(W)$

$n = W.rows$
let $\Pi = (\pi_{ij})$ be a new $n \times n$ matrix
for $i = 1$ **to** n
    for $j = 1$ **to** n
        if $i == j$ **or** $w_{ij} == \infty$
            $\pi_{ij} =$ NIL
        else
            $\pi_{ij} = i$
$D = W$
for $k = 1$ **to** n
    for $i = 1$ **to** n
        for $j = 1$ **to** n
            if $d_{ij} > d_{ik} + d_{kj}$
                $\pi_{ij} = \pi_{kj}$
            $d_{ij} = \min(d_{ij}, d_{ik} + d_{kj})$
return D, Π

Does this calculate the predecessor matrix correctly? That is, does the augmented FLOYD-WARSHALL$'$ return the same Π matrix as the augmented FLOYD-WARSHALL does? In explaining your answer, keep in mind that values from the D matrix are used in computing values for the Π matrix, so your reasoning needs to reflect that superscripts were dropped from both.

Exercise 27.1-x1 (3 points)

1. Rewrite the FLOYD-WARSHALL$'$ algorithm from Exercise 25.2-4 (p. 699) with as many of the **for** loops changed to **parallel for** as is legal.

2. What is the work, span, and parallelism for your modified algorithm? Give your answers as Θ expressions in terms of n.